A generalized stabilization scheme for seismic Q compensation

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Outline

Motivation

k-spcae Green's function

Generalized stabilization scheme

Parameters selection

Q-RTM exmaples

Discussion and conclusions

Motivation

- Amplitude attenuation and phase distortion caused by the anelasticity of subsurface media degrade the quality of migrated images and the reliability of the subsequent interpretation.
- A common issue existing in seismic Q compensation is the numerical instability. It has been stated in the literature that direct amplitude compensation will inevitably result in exponentially boosted high-frequency noise.
- We provide a brief overview of several widely used stabilization strategies for seismic Q compensation.

Motivation

- We start from our previous work of deriving the k-space Green's function for the compensated constant-Q wave equation.
- We then propose a generalized stabilization scheme for seismic Q compensation by incorporating a window function into the exponentially divergent time propagator.
- With an assumption that the exponent of the chosen window is a power function of the magnitude of wavenumber, we formulate an explicit stabilization term for the Q-compensated constant-Q wave equation in the time-space domain.

Motivation

Explicit stabilization may have following advantages over the implicit schemes:

1. More convenient workflow for seismic Q compensation in time-space domain.
2. No FFT needed when FDM is available for calculating fractional Laplacians.
3. Relatively greater tolerance for parameter selection.
4. Precise phase correction.
5. Physically clear meaning of regularization.

Outline

Constant Q wave equation

We consider the constant-Q viscoacoustic wave equation with decoupled fractional Laplacians (DFLs), which is firstly proposed by[1] as follow:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \eta(-\nabla^2)^{\gamma+1}p - \tau\frac{\partial}{\partial t}(-\nabla^2)^{\gamma+1/2}p = \delta(\mathbf{x}_s)f(t), \quad (1)$$

where the coefficients before two fractional Laplacians, separately stands for dispersion and absorption, are given by $\eta = -c_0^{2\gamma}\omega_0^{-2\gamma}\cos(\pi\gamma)$ and $\tau = -c_0^{2\gamma-1}\omega_0^{-2\gamma}\sin(\pi\gamma)$.

[1] Tieyuan Zhu and Jerry M. Harris. "Modeling acoustic wave propagation in heterogeneous attenuating media using decoupled fractional Laplacians". In: *Geophysics* 79.3 (2014), T105–T116.

Compensated constant Q wave equation

The compensated constant-Q wave equation can be achieved by reversing the absorption coefficient in sign but leaving the equivalent dispersion parameter unchanged[2,3], which is given by

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \eta(-\nabla^2)^{\gamma+1}p + \tau\frac{\partial}{\partial t}(-\nabla^2)^{\gamma+1/2}p = \delta(\mathbf{x}_r)g(\mathbf{x}, T-t). \tag{2}$$

[2] Bradley E Treeby, Edward Z Zhang, and B T Cox. "Photoacoustic tomography in absorbing acoustic media using time reversal". In: *Inverse Problems* 26 (2010), pp. 115003–20.

[3] Tieyuan Zhu, Jerry M. Harris, and Biondo Biondi. "Q-compensated reverse-time migration". In: *Geophysics* 79.3 (2014), S77–S87.

k-space Green's function

According to our previous work[4], the k-space Green's function of the compensated equation 2 is

$$G(\mathbf{k},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{i\omega(t-t_0)}e^{-i\mathbf{k}(\mathbf{x}-\mathbf{x}_s)}}{\frac{\omega^2}{c^2} + \eta|\mathbf{k}|^{2\gamma+2} - i\omega\tau|\mathbf{k}|^{2\gamma+1}} d\omega. \tag{3}$$

The integral kernel function in this equation has two singularities which can be obtained by solving ω for the following equation:

$$\frac{\omega^2}{c^2} + \eta|\mathbf{k}|^{2\gamma+2} - i\omega\tau|\mathbf{k}|^{2\gamma+1} = 0. \tag{4}$$

[4]YF Wang et al. "The K-Space Green's Functions for Decoupled Constant-Q Wave Equation and its Adjoint Equation". In: *79th EAGE Conference and Exhibition 2017*. 2017.

k-space Green's function

Equation 4 is also known as the dispersion relation of the compensated constant-Q wave equation, and its solutions are given by

$$\zeta_{1,2}(\mathbf{k}) = \pm\xi_1(\mathbf{k}) - i\xi_2(\mathbf{k}), \tag{5}$$

where $\xi_1(\mathbf{k}) = \frac{1}{2}\sqrt{-\tau^2c^4|\mathbf{k}|^{4\gamma+2} - 4\eta c^2|\mathbf{k}|^{2\gamma+2}}$ and $\xi_2(\mathbf{k}) = -\frac{1}{2}\tau c^2|\mathbf{k}|^{2\gamma+1} > 0$ are absolute real part and imaginary part of the solution, respectively. The time propagator of the compensated equation 2, is given by[5]

$$\Gamma_{comp}(\mathbf{k}, t) = G(\mathbf{k}, t) = \frac{\sin(\xi_1(\mathbf{k})t)e^{\xi_2(\mathbf{k})t}}{\xi_1(\mathbf{k})}. \tag{6}$$

[5] Yufeng Wang et al. "Adaptive stabilization for Q-compensated reverse time migration". In: *Geophysics* 83.1 (2018), S15–S32.

Outline

Window function

We incorporate a window function $e^{\phi(\mathbf{k})t}$ into the exponentially divergent time propagator. Thus, the stabilized time propogator can be expressed as

$$\Gamma_{sta}(\mathbf{k}, t) = \frac{\sin(\xi_1(\mathbf{k})t)e^{\xi_2(\mathbf{k})t}e^{\phi(\mathbf{k})t}}{\xi_1(\mathbf{k})}, \tag{7}$$

where the window function $e^{\phi(\mathbf{k})t}$ aims at suppressing the high-wavenumber component of the time propogator.

Window function

Mathematically, the conventional low-pass filtering method and our previously proposed adaptive stabilization approach can be taken as a special case of the generalized stabilization framework. More specifically, equation 7 represents low-pass filtered time propogator if we assume that $e^{\phi(\mathbf{k})t}$ is a Tukey window function; equation 7 reduces to an adaptive stabilized time propogator if we assume that $e^{\phi(\mathbf{k})t}$ satisfies

$$e^{\phi(\mathbf{k})t} = \frac{1}{1 + \sigma^2 e^{2\xi_2(\mathbf{k})t}}, \tag{8}$$

where σ^2 is the stabilization factor.

Exponential window

However, neither the Tukey window function nor the adaptive stabilization operator can track back to an expilict stabilization term in time-space domain. Luckily, if we assume that the exponent of the chosen window is a power function of the magnitude of wavenumber, we can derive a Q-compensated equation with an expilict stabilization term, which is desirable particularly for Q-RTM. We consider

$$\phi(\mathbf{k}) = -\sigma^2 |\mathbf{k}|^\alpha, \tag{9}$$

where the order α is typically an integer, the negative coefficient $-\sigma^2$ aims to achieve a trade-off between fedility and stability.

Backtracking

Substituting equation 9 into equation 7, we have

$$\Gamma_{sta}(\mathbf{k},t) = \frac{\sin(\xi_1(\mathbf{k})t)e^{\left(\xi_2(\mathbf{k})-\sigma^2|\mathbf{k}|^\alpha\right)t}}{\xi_1(\mathbf{k})}. \tag{10}$$

Then we can track back to equation 5, the solutions of the stabilized dispersion relation are

$$\zeta'_{1,2}(\mathbf{k}) = \pm\xi'_1(\mathbf{k}) - i\xi'_2(\mathbf{k}), \tag{11}$$

where $\xi'_2(\mathbf{k}) = \xi_2(\mathbf{k}) - \sigma^2|\mathbf{k}|^\alpha = -\frac{1}{2}\tau c^2\left(|\mathbf{k}|^{2\gamma+1} + \frac{2\sigma^2}{\tau c^2}|\mathbf{k}|^\alpha\right)$.

Backtracking

We need to be aware that the real part $\xi_1'(\mathbf{k})$ and the imaginary part $\xi_2'(\mathbf{k})$ are related to each other. Now that we modify the ξ_2 into ξ_2', $\xi_1'(\mathbf{k})$ is no longer the same as $\xi_1(\mathbf{k})$. From equation 6, we find that $\xi_1(\mathbf{k})$ mainly controls the phase of the time propogator, whereas ξ_2 mainly determines the amplitude of the time propogator. This observation inspires us to keep ξ_1 unchanged.

Backtracking

We introduce a function $\psi(\mathbf{k})$ satisfying the following relation

$$\xi_1'(\mathbf{k}) = \frac{1}{2}\sqrt{-\tau^2 c^4\left[|\mathbf{k}|^{2\gamma+1} + \frac{2\sigma^2}{\tau c^2}|\mathbf{k}|^{\alpha}\right]^2 - 4\eta c^2\left[|\mathbf{k}|^{2\gamma+2} + \psi(\mathbf{k})\right]}$$
$$= \frac{1}{2}\sqrt{-\tau^2 c^4|\mathbf{k}|^{4\gamma+2} - 4\eta c^2|\mathbf{k}|^{2\gamma+2}} = \xi_1(\mathbf{k}). \quad (12)$$

Then we can solve $\psi(\mathbf{k})$ in this equation, it gives

$$\psi(\mathbf{k}) = \frac{-\tau c^2\sigma^2|\mathbf{k}|^{2\gamma+1+\alpha} - \sigma^4|\mathbf{k}|^{2\alpha}}{\eta c^2}. \quad (13)$$

Backtracking

So far we have known two solutions of the stabilized dispersion relation, thus, we can construct this dispersion relation as follow:

$$\frac{\omega^2}{c^2} + \eta\left(|\mathbf{k}|^{2\gamma+2} - \frac{\tau\sigma^2}{\eta}|\mathbf{k}|^{2\gamma+1+\alpha} - \frac{\sigma^4}{\eta c^2}|\mathbf{k}|^{2\alpha}\right) - i\omega\tau\left(|\mathbf{k}|^{2\gamma+1} + \frac{2\sigma^2}{\tau c^2}|\mathbf{k}|^{\alpha}\right) = 0. \tag{14}$$

Stabilized wave equation with Q compensation

Transforming equation 14 back to time-space domain, we obtain the stabilized wave equation with Q compensation

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \eta(-\nabla^2)^{\gamma+1}p + \tau\frac{\partial}{\partial t}(-\nabla^2)^{\gamma+1/2}p + \tau\sigma^2(-\nabla^2)^{(2\gamma+1+\alpha)/2}p + \frac{\sigma^4}{c^2}(-\nabla^2)^{\alpha}p + \frac{2\sigma^2}{c^2}\frac{\partial}{\partial t}(-\nabla^2)^{\alpha/2}p = 0. \tag{15}$$

Stabilized wave equation with Q compensation

We sometimes omit the phase correction term due to the numerical dispersion is relatively not severer than amplitude attenuation.

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \eta(-\nabla^2)^{\gamma+1}p + \tau\frac{\partial}{\partial t}(-\nabla^2)^{\gamma+1/2}p + \tau\sigma^2(-\nabla^2)^{(2\gamma+1+\alpha)/2}p + \frac{\sigma^4}{c^2}(-\nabla^2)^{\alpha}p + \frac{2\sigma^2}{c^2}\frac{\partial}{\partial t}(-\nabla^2)^{\alpha/2}p = 0. \tag{16}$$

Tracking and Backtracking

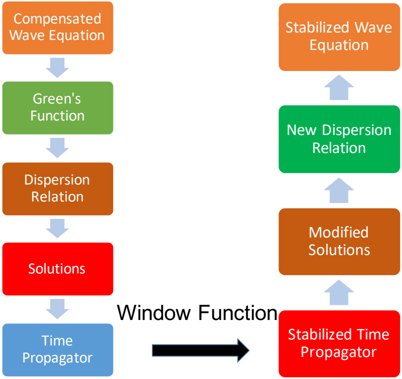

Figure 0. The flow of tracking and backtracking.

Outline

Stabilization order α

There are two criteria to determine stabilization order α,

- the filtering property of exponential window function;
- the computational cost of the explicit stabilization term (if we omit the phase correction term).

Obviously, the first criterion owns priority over the second one.

Stabilization order α

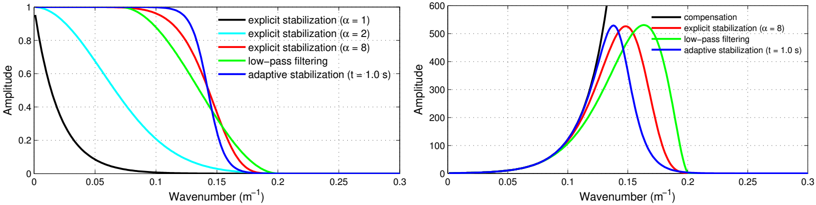

Figure 1. (a) Different window functions for stabilization and (b) the stabilized compensation curves with window functions.

Stabilization factor σ^2

Stabilization factor σ^2 is an another important parameter affecting the filtering performance of the window functions. Let we have a closer look on equation 7, coefficient σ^2 controls the contribution of the compensation term $|\mathbf{k}|^{2\gamma+1}$ and the stabilization term $\frac{2\sigma^2}{\tau c^2}|\mathbf{k}|^{\alpha}$ (where $\tau < 0$). Thus, stabilization factor σ^2 should be carefully chosen to ensure a comparable magnitude between these two terms.

Stabilization factor σ^2

We introduce a reference scaling factor S_{ref} to facilitate estimation of σ^2.

$$S_{ref} = \frac{-\frac{1}{2}\tau c^2 |\mathbf{k}_{ref}|^{2\gamma+1}}{|\mathbf{k}_{ref}|^{\alpha}}, \quad (17)$$

where $\mathbf{k}_{ref}$ is typically close to the maximum wavenumber, i.e., $\mathbf{k}_{ref} = 2\pi/(2d_x)$. Then stabilization factor σ^2 can be roughly determined within a range of $(0.5S_{ref}, 2S_{ref})$.

Stabilization factor σ^2

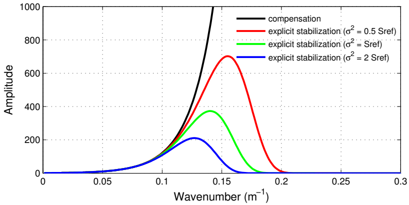

Figure 2. The explicit stabilization with different factors σ^2.

Outline

Marmousi model

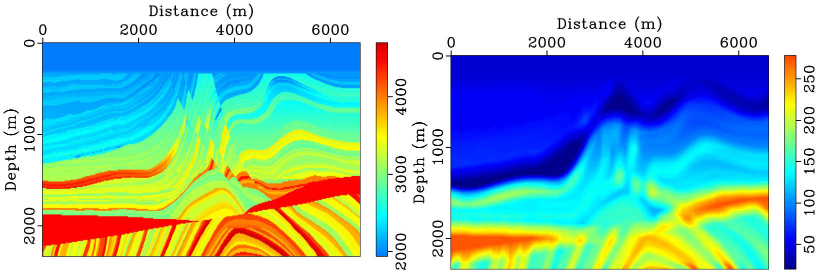

Figure 3. (a) Velocity and (b) Q of the Marmousi model.

Q-RTM images

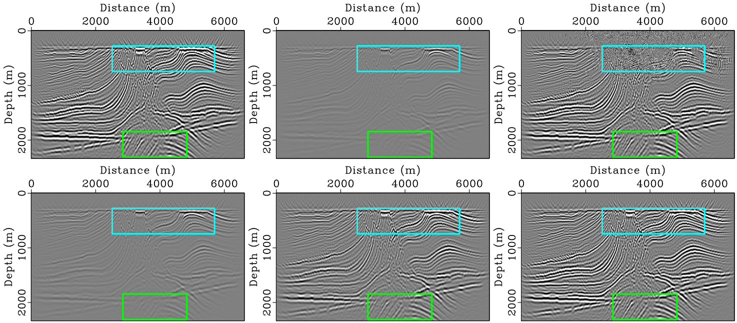

Figure 4. Seismic imaging results of the Marmousi model using (a) acoustic RTM on lossless data, (b) acoustic RTM on lossy data, (c) Q-RTM without stabilization, (d) Q-RTM with $\alpha = 1$, (e) Q-RTM with $\alpha = 2$, and (f) Q-RTM with $\alpha = 8$. We set $\sigma^2 = S_{ref}$ for (d), (e) and (f).

Q-RTM traces

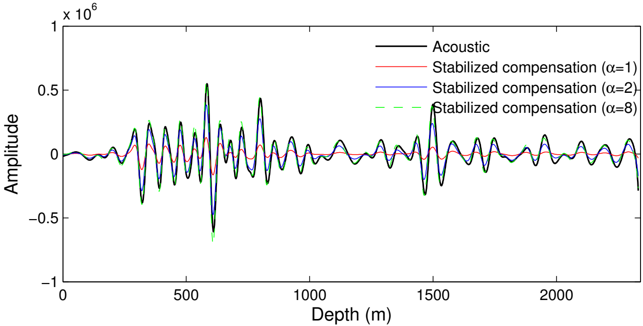

Figure 4. The traces selected at a distance of 4000 m from imaging results.

Outline

Discussion

- The proposed explicit stabilization scheme has potential to provide a more convenient workflow for seismic Q compensation. It enables us to be free of the third-party library and Fourier transform needed for low-pass filtering functions when finite difference algorithms are available for calculating fractional Laplacians.
- Although we have derived a phase correction term in the stabilized wave equation 15, but we simply omit this term in this paper, we would like to explore the effect of this term in our future work.

Conclusions

- Under an assumption that the exponent of the chosen window is a power function of the magnitude of wavenumber, we formulated an explicitly stabilized wave equation with both phase correction term and stabilization term in the time-space domain.
- We provided a robust method for parameters selection by introducing a reference scaling factor. Q-RTM tests on synthetic data further verified the feasibility and stability of the proposed method.

References:

- Zhu, Tieyuan and Jerry M. Harris. "Modeling acoustic wave propagation in heterogeneous attenuating media using decoupled fractional Laplacians". In: *Geophysics* 79.3 (2014), T105–T116.
- Treeby, Bradley E, Edward Z Zhang, and B T Cox. "Photoacoustic tomography in absorbing acoustic media using time reversal". In: *Inverse Problems* 26 (2010), pp. 115003–20.
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Wang, Yufeng et al. "Adaptive stabilization for Q-compensated reverse time migration". In: *Geophysics* 83.1 (2018), S15–S32.

Thank you!